Translating Word Problems: Keywords *(page 1 of 2)*

Sections: Keywords, Worked examples

The hardest thing about doing word problems is taking the English words and translating them into mathematics. Usually, once you get the math equation, you're fine; the actual math involved is often fairly simple. But figuring out the actual equation can seem nearly impossible. What follows is a list of hints and helps. Be advised, however: To *really* learn "how to do" word problems, you will need to practice, practice, practice.

The first step to effectively translating and solving word problems is to read the problem entirely. Don't start trying to solve anything when you've only read half a sentence. Try first to get a feel for the whole problem; try first to see what information you have, and what you still need.

The second step is to work in an organized manner. Figure out what you need but don't have, and name things. Pick variables to stand for the unknows, clearly labelling these variables with what they stand for. Draw and label pictures neatly. Explain your reasoning as you go along. And make sure you know just exactly what the problem is actually asking for. You need to do this for two reasons:

1. Working clearly will help you think clearly, and
2. figuring out what you need will help you translate your final answer back into English.

Regarding (2) above: It can be really frustrating (and embarassing) to spend fifteen minutes solving a word problem on a test, only to realize at the end that you no longer have any idea what "x" stands for, so you have to do the whole problem over again. I did this on a calculus test -- thank heavens it was a short test! -- and, trust me, you don't want to do this to yourself!

The third step is to look for "key" words. Certain words indicate certain mathematical operations. Below is a partial list.

Addition	increased by more than combined, together total of sum added to
Subtraction	decreased by minus, less difference between/of less than, fewer than
Multiplication	of times, multiplied by product of increased/decreased by a factor of (this type can involve both addition or

	subtraction *and* multiplication!)
Division	per, a out of ratio of, quotient of percent (divide by 100)
Equals	is, are, was, were, will be gives, yields sold for

Note that "per" means "divided by", as in "I drove 90 miles on three gallons of gas, so I got 30 miles per gallon". Also, "a" sometimes means "divided by", as in "When I tanked up, I paid $\$12.36$ for three gallons, so the gas was $\$4.12$ a gallon".

Warning: The "less than" construction is backwards in the English from what it is in the math. If you need to translate "1.5 less than x", the temptation is to write "$1.5 - x$". *Do not do this!* You can see how this is wrong by using this construction in a "real world" situation: Consider the statement, "He makes $\$1.50$ an hour less than me." You do not figure his wage by subtracting your wage from $\$1.50$. Instead, you subtract $\$1.50$ from your wage. So remember; **the "less than" construction is backwards.**

Also note that order is important in the "quotient/ratio of" and "difference between/of" constructions. If a problems says "the ratio of x and y", it means "x divided by y", not "y divided by x". If the problem says "the difference of x and y", it means "$x - y$", not "$y - x$".

Now we need to learn to extract the keywords from the word problems....

Translating Word Problems: Examples *(page 2 of 2)*

Sections: Keywords, Worked examples

- **Translate "the sum of 8 and y" into an algebraic expression.**

  This translates to "$8 + y$"

- **Translate "4 less than x" into an algebraic expression.**

  This translates to "$x - 4$"

Remember? "Less than" is *backwards* in the math from how you say it in words!

- **Translate "x multiplied by 13" into an algebraic expression.**

  This translates to "$13x$"

- **Translate "the quotient of x and 3" into an algebraic expression.**

  This translates to "$x/3$"

- **Translate "the difference of 5 and y" into an algebraic expression.**

  This translates to "$5 - y$"

- **Translate "the ratio of 9 more than x to x" into an algebraic expression.**

  This translates to "$(x + 9) / x$"

- **Translate "nine less than the total of a number and two" into an algebraic expression, and simplify.**

  This translates to "$(n + 2) - 9$", which then simplifies to "$n - 7$"

Here are some more wordy examples:

- **The length of a football field is 30 yards more than its width. Express the length of the field in terms of its width w.**

  Whatever the width w is, the length is 30 more than this. Recall that "more than" means "plus that much", so you'll be adding 30 to w.

  The expression they're looking for is "$w + 30$".

This one is important:

- **Twenty gallons of crude oil were poured into two containers of different size. Express the amount of crude oil poured into the smaller container in terms of the amount g poured into the larger container.**

  The expression they're looking for is found by this reasoning: There are twenty gallons total, and we've already poured g gallons of it. How many gallons are left? There are $20 - g$ gallons left. They want the answer "$20 - g$".

This is the "how much is left" construction: You will be given some total amount. Smaller amounts, of unspecified sizes, are added (combined, mixed, etc) to create this total amount. You will pick a variable to stand for one of these unknown amounts. After having thus accounted for one of the amounts, the remaining amount is whatever is left after deducting this named amount from the total.

- They may tell you that a trip took ten hours, and that the trip had two legs. You might name the time for the first leg as $"t"$, with the *remaining* time for the second leg being $10 - t$.
- They may tell you that a hundred-pound order of animal feed was filled by mixing products from Bins A, B, and C, and that twice as much was added from Bin C as from Bin A. Let $"a"$ stand for the amount from Bin A. Then the amount from Bin C was $"2a"$, and the amount taken from Bin B was the remaining portion of the hundred pounds: $100 - a - 2a$.

I'm making a big deal about this "how much is left" construction because it comes up a lot and tends to cause a lot of confusion. Make sure you understand this one!

Once you've learned to translate phrases into expressions and sentences into equations, you are ready to dive into word problems. Of course, there are infinitely-many possible word problems (physics is all word problems; business math is all word problems; "real life" can feel like an essay question...). The following links lead to explanations and examples of some basic types of word problems that you can expect to see in your classes:

"Age" problems, involving figuring out how old people are, were, or will be
"Area/volume/perimeter" problems, involving very basic geometric formulas
"Coin" problems, involving figuring out how many of each type of coin you have
"Distance" problems, involving speeds ("uniform rates"), distance, time, and the formula "$d = rt$".
"Investment" problems, involving investments, interest rates, and the formula "$I = Prt$".
"Mixture" problems, involving combining elements and find prices (of the mixure) or percentages (of, say, acid or salt).
"Number" problems, involving "Three more than two times the smaller number..."
"Percent of" problems, involving finding percents, increase/decrease, discounts, etc.
Quadratic word problems, such as projectile motion and max/min questions.
"Work" problems, involving two or more people or things working together to complete a task, and finding how long they took.